Sensitivity of multi-product two-stage economic lotsizing models and their dependency on change-over and product cost ratio's

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Abstract

This study considers the production and inventory management problem of a two-stage semi-process production system. In case both production stages are physically connected it is obvious that materials are forced to flow. The economic lotsize depends on the holding cost of the end-product and the combined change-over cost of both production stages. On the other hand this 'flow shop' is forced to produce at the speed of the slowest stage. The benefit of this approach is the low amount of Work In Process inventory. When on the other hand, the involved stages are physically disconnected, a stock of intermediates acts as a decoupling point. Typically for the semi-process industry are high change-over costs for the process oriented first stage, which results in large lotsize differences for the different production stages. Using the stock of intermediates as a decoupling point avoids the complexity of synchronising operations but is an additional reason to augment the intermediate stock position. The disadvantage of this model is the high amount of Work-In-Process inventory.

This paper proposes the 'synchronised planning model' realising a global optimum instead of the combination of two locally optimised settings. The mathematical model proves (for a two-stage single-product setting) that the optimal two-stage production frequency corresponds with the single EOQ solution for the first stage. A sensitivity study reveals, within these two-stage lotsizing models, the economical cost dependency on product and change-over cost ratio's. The purpose of this paper is to understand under which conditions the 'joined setup' or the 'two-stage individual eoq model' remain close to the optimal model. Numerical examples prove that the conclusions about the optimal settings remain valid when extending the model to a two-stage multi-product setting. The research reveals that two-stage individually optimized EOQ lotsizing should only be used when the end-product stage has a high added value and small change-over costs, compared to the first stage. Physically connected operations should be used when the end-product stage has a small added value and low change-over costs, or high added value and large change-over costs compared to the first production stage.

The paper concludes with suggesting a practical common cycle approach to tackle a two-stage multi-product production and inventory management problem. The common cycle approach brings the benefit of a repetitive and predictable production schedule.

Keywords: *Economical Lotsizing, Two-stage production system, Cyclical Production Scheduling, Economic Lot Scheduling Problem (ELSP).*

1. Introduction

This research paper focuses on the semi-process industry and more specific on the question of economical lotsizing decisions for a two-stage production setting. Within such a Make-To-Stock environment management faces following dilemmas:

In case different production stages are physically connected it is obvious that materials are forced to flow. The economic lotsize depends on the holding cost of the end-product and the combined change-over cost of both production stages. (joined setup model). On the other hand this 'flow shop' is forced to produce at the speed of the slowest stage. Furthermore a breakdown of one of the involved stages causes an interrupt for the complete production process. The benefit of this approach is the low amount of Work-In-Process inventory.

When on the other hand, the involved stages are physically disconnected, a stock of intermediates acts as a decoupling point. Typically for the semi-process industry set-up costs for the process oriented part are high, which results in large lotsize differences for the different production stages (two-stage individual eoq model). Based on these economical lotsize considerations, the size of the intermediate stock is higher than the buffer stock needed to feed the second stage process. Most often stage one and two are different departments or organisations, both trying to 'locally' optimise their operations. Using the stock of intermediates as a decoupling point avoids the complexity of synchronising operations but is an additional reason to augment the intermediate stock position. The disadvantage of this model is the high amount of Work-In-Process inventory.

This paper proposes the 'synchronised planning model' bringing a global optimum instead of the combination of two locally optimised settings. In order to understand the dynamics of the system this research defines the (mathematically) optimal production frequencies at both production stages (optimal two-stage lotsizing model). The main benefit of this approach, still minimising the inventory and set-up cost function, is the ability to reduce the amount of Work In Process inventories.

A sensitivity study should reveal, within these two-stage lotsizing models, the economical cost dependency on product and change-over cost ratio's. The purpose of this paper is to understand under which conditions the 'joined setup' or the 'two-stage individual eoq model' remain close to the optimal model.

This paper has following structure. Section 1 introduces the problem.. Section 2 highlights on previous research considering multi-echelon lotsizing models. Section 3 describes the studied two-stage lotsizing models, introduces the 'synchronised planning model' and performs, within a single-product setting, the sensitivity study for different setup and holding cost ratio's. Section 4 extends the single product model to a multi-product model, having one first stage product and multiple second stage products. Section 5 suggests a practical cyclical planning approach to tackle a two-stage multi-product production and inventory management problem. Section 6 concludes.

2. Previous Research

Pioneering work on multi-echelon lot size models has been done by Crowston and Wagner (1973). Under EOQ like assumptions of constant continuous demand, instantaneous production, zero lead times, infinite planning horizon, fixed setup costs and linear holding costs they prove that for a multi-stage assembly system, the optimal lotsize at each facility is an integer multiple of the lotsize at the successor facility.

In order to synchronise operations, researchers have been interested in determining cyclical production schedules for multiple products over an infinite planning horizon , minimising the sum of setup and inventory holding costs. Two techniques, assuming static demand, are based on cyclic scheduling and solve the multi-product Economic Lot Scheduling Problem (ELSP) within a capacity constrained environment.

The 'common cycle' approach, attributed to Hanssmann (1962) solves the model mathematically and determines the optimal length of a production cycle where each product is produced at the same common base frequency. A generalisation of this model towards a multi-stage environment is described by El-Najdawi (1993).

The 'cyclic scheduling' approach is an extension of the common cycle model allowing schedules in which each product may be produced more than once during the base cycle time. This approach is described as the 'basic period approach'. The production frequency of each product is expressed as an integer multiple of a common basic period. The fact that the common cycle method is more restricted (allowing only frequency factors equal to one for each involved product) explains why the common cycle (analytical) solution is an upper bound for the ELSP problem (Doll & Whybark 1973). The Economic Lot Scheduling Problem solution, allowing different frequency factors, results in a plan with lower inventory and set-up costs. The set of independent, EOQ based, solutions on the other hand represents the lower bound. In the early seventies a number of authors (Doll & Whybark(1973), Goyal(1973)) presented heuristic approaches to solve the ELSP problem. Doll & Whybark (1973) present a heuristic solution and solve the multi-product single-stage lotsizing problem by determining for each product a production frequency equal to an integer multiple of a basic period. Elmaghraby (1978) reviews the different techniques for solving the ELSP problem, and indicates the ability of the proposed heuristics to present a good and acceptable solution. All compared heuristics (Doll& Whybark, Goyal) are superior in their performance to the analytical approaches. This is mainly due to the restrictive assumptions made in these analytical models.

Davis (1995) proposes a decomposition two-phase approach to solve the ELSP problem and sequence the proposed lotsizes. This two-phase process, combining a heuristic procedure and mixed integer programming, resolves the combinatorial difficulties of jointly determining production quantities, frequencies and processing sequences. The first phase defines the basic period and production frequencies. The scheduling sequence is determined by the second phase process.

The benefits of common repetitive production plan for multiple items are numerous. The predictability of the schedule allows synchronisation between different production stages. In situations where the capacity resource constraint is tight a cyclical model, evenly spreading the load on operations, reduces the possibility of peak demands arriving all at once and claiming the use of the capacity constrained resource. Simulation results suggest that the fixed cycle approach, shows greater improvement over the independent cycle approach in capacity constrained situations (Güder and Zydiak 2000). The implied cyclical material coordination aspects result in a more robust planning stability (Van Donselaer et al 2000).

When extending this model towards a multi-stage process, Viswanathan and Piplani (2001) demonstrate that using common replenishment periods can establish a co-ordinated inventory policy and brings substantial savings for the total system, in comparison with the independent cycle approach. These savings appear when the involved set-up costs are above a certain threshold level. The process with the largest set-up cost dictates the behaviour of the potential savings. When on the other hand set-up costs are of low importance a flow production should be more adequate.

Van den broecke et al (2008), describe a practical application within a photographic film producing company, implementing a near-optimal solution for the multi-stage, multi-product capacitated lot-sizing problem by rolling out a cyclical production plan.

3. Single-product two-stage model

Within the two-stage single product setting, D corresponds with the annual demand. The first stage uses a lotsize Q = D/M, where M equals the annual production frequency. The second stage production lotsize corresponds with q = Q/k where k is the frequency difference factor between first and second production stage. On an annual basis the first and second stage count respectively M and M.k production runs.

3.1 Optimal two-stage lotsizing model

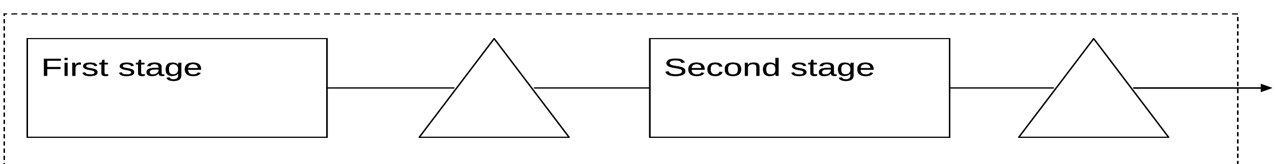

Fig 1 : Optimal two-stage lotsizing model

Using the model's convention (fig. 1) the second stage lotsize corresponds with q = D/(M.k). Assuming constant demand the average end-product stock corresponds with D/(2.M.k). Within the synchronized planning model, in reaction on the first stage delivery of D/M volume of intermediate product, the second stage process immediately converts q = D/(M.k) units into end-product. Based on this synchronisation 'push' concept, the average intermediate stock corresponds with (Q-q)/2 = [D.M – D/(M.k)]/2 = [D/(2.M)](1-1/k).

Within the EOQ logic S and s stand for setup costs within first and second production stage. P and p_a are product costs of both stages and h stands for the inventory holding cost percentage. Total annual setup and inventory cost (TC) equals :

$$TC = M.S + \frac{D}{2M}(1-\frac{1}{k})h.P + s.k.M + \frac{D}{2.k.M}.h.p = M.S + s.k.M + \frac{D}{2.M}.h.P + \frac{D}{2.k.M}.h.(p-P)$$

Taking the derivatives to both M and k equal to zero defines the position of the optimum.

Derivative to k :

$$s.M - (\frac{1}{k^2}).\frac{D}{2M}.h.(p-P) = 0$$

$$k^2 = \frac{D.h.(p-P)}{2.s.M^2}$$

$$k = \frac{1}{M}\sqrt{\frac{D.h.(p-P)}{2.s}}$$

$$k = \frac{\sigma}{M} \qquad with\ \sigma = \sqrt{\frac{D.h.(p-P)}{2.s}}$$

Derivative to M :

$$S + s.k - \frac{D.h.P}{2.M^2} - \frac{D.h.(p-P)}{2.k.M^2} = 0$$

$$S + s.\frac{\sigma}{M} - \frac{D.h.P}{2.M^2} - \frac{D.h.M(p-P)}{2.\sigma.M^2} = 0$$

$$S + s.\frac{\sigma}{M} - \frac{D.h.P}{2.M^2} - \frac{D.h.M(p-P)}{2.\sigma.M^2} = 0$$

$$M^2.S + M.(s.\sigma - \frac{D.h.(p-P)}{2.\sigma}) - \frac{D.h.P}{2} = 0$$

$$M^2.S - \frac{D.h.P}{2} = 0 \ (*)$$

$$M^2 = \frac{D.h.P}{2.S}$$

$$M = \sqrt{\frac{D.h.P}{2.S}}$$

$$s.\sigma - \frac{D.h.(p-P)}{2.\sigma} = s_a\sqrt{\frac{D.h.(p-P)}{2.s_a}} - \frac{D.h.(p-P)}{2.\sqrt{\frac{D.h.(p-P)}{2.s}}} =$$

$$\sqrt{\frac{s.D.h.(p-P)}{2}} - \frac{D.h.(p-P).\sqrt{s}}{\sqrt{2.D.h.(p-P)}} = \sqrt{\frac{s.D.h.(p-P)}{2}} - \sqrt{\frac{sD.h.(p-P)}{2}} = 0$$

(*) Replacing σ with its value eliminates the first grade term.

The mathematical calculus proves that the first stage optimal production rate M for the two-stage system equals the single-stage EOQ (first stage) production frequency. The optimal values of M and k correspond with :

$$M = \sqrt{\frac{D.h.P}{2.S}} \qquad k = \frac{1}{M}.\sqrt{\frac{D.h.(p-P)}{2.s}} = \frac{\sqrt{2.S}}{\sqrt{D.h.P}}.\frac{\sqrt{D.h.(p-P)}}{\sqrt{2.s}} = \sqrt{\frac{(p-P).S}{P.s}}$$

Within the sensitivity study of the cost function for various ratios of S/s (setup cost ratio) and P/p (product cost ratio) , this paper assumes a first stage setup cost of 5528 euro and a first stage product cost of 1 euro/sqm. The annual demand equals 17,244 million sqm. These data correspond with the actual parameter settings for a medical X-ray filmtype (LT2B type) within a photographic film producing company. The inventory holding percentage is set at 16%.

Table 1 and 2 illustrate the different settings of setup and product cost ratio's and the corresponding cost function. For the highlighted example , the ratio (p-P)/P = 2 implies a second stage product cost of p = 3 euro/sqm and the ratio S/s = 1.5 implies a second stage setup cost of s = 3685 euro. Within this example the 'optimal two-stage lotsizing model' corresponds with M = 15.7972, k = 1.7321 resulting in a total cost function equal to 376327 euro. Table 1 illustrates for various settings of p and s, the resulting (optimal) k values. Table 2 lists the corresponding total cost function. Since values of k < 1 are not allowed the optimal total cost is only presented for valid k conditions.

k	S/s								
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50	0.5000	0.7071	0.8660	1.0000	1.1180	1.2247	1.3229	1.4142	1.5000	1.5811
1.00	0.7071	1.0000	1.2247	1.4142	1.5811	1.7321	1.8708	2.0000	2.1213	2.2361
1.50	0.8660	1.2247	1.5000	1.7321	1.9365	2.1213	2.2913	2.4495	2.5981	2.7386
2.00	1.0000	1.4142	1.7321	2.0000	2.2361	2.4495	2.6458	2.8284	3.0000	3.1623
2.50	1.1180	1.5811	1.9365	2.2361	2.5000	2.7386	2.9580	3.1623	3.3541	3.5355
3.00	1.2247	1.7321	2.1213	2.4495	2.7386	3.0000	3.2404	3.4641	3.6742	3.8730
3.50	1.3229	1.8708	2.2913	2.6458	2.9580	3.2404	3.5000	3.7417	3.9686	4.1833
4.00	1.4142	2.0000	2.4495	2.8284	3.1623	3.4641	3.7417	4.0000	4.2426	4.4721

Table 1 : k-values for the optimal two-stage lotsizing model

TC	S/s									
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50				261981	252761	245956	240667	236403	232872	229884
1.00		349308	317258	298153	285115	275490	268010	261981	256986	252761
1.50		388560	349308	325908	309940	298153	288992	281607	275490	270316
2.00	523961	421652	376327	349308	330869	317258	306680	298153	291090	285115
2.50	565192	450806	400131	369923	349308	334090	322263	312730	304833	298153
3.00	602466	477163	421652	388560	365977	349308	336352	325908	317258	309940
3.50	636744	501401	441442	405699	381307	363301	349308	338027	328684	320780
4.00	668649	523961	459862	421652	395575	376327	361367	349308	339319	330869

Table 2 : Total Cost within the optimal two-stage lotsizing model

3.2 Joined setup EOQ model

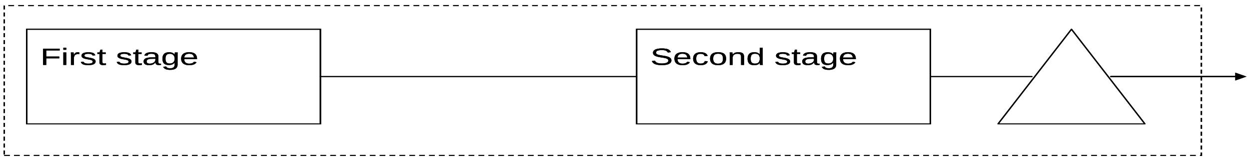

Fig 2 : Joined setup EOQ model

Within the 'Joined setup EOQ model', both stages have the same lotsize Q and produce at the same rate (implying k = 1). Since both stages are physically connected there is no intermediate stock and the economical production lotsize is controlled by the end-product holding cost and the sum of the first and second stage setup costs. The lotsize and total annual cost formulate as :

$$Q = \sqrt{\frac{2.D.(S+s)}{h.p}} \qquad M = \frac{D}{Q} = \sqrt{\frac{D.h.p}{2.(S+s)}} \qquad k = 1 \qquad TC = M.(S+s) + \frac{D.h.p}{2.M}$$

Within the sensitivity analysis, total cost is calculated for various settings of s and p (table 3). The comparison of the 'joined setup' total cost function with the 'optimal total cost' only takes place for the valid (s , p) settings of the 'optimal two-stage lotsizing model' (table 4). Cost deviations to the optimal model can be quite substantial (> 20 %) especially in the case of high added value by the second production stage (high value for (p-P)/P) and large setup costs within the first production stage (high value for S/s).

TC	S/s									
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50	370497	302509	276152	261981	253097	246998	242547	239155	236482	234323
1.00	427813	349308	318873	302509	292252	285208	280069	276152	273066	270572
1.50	478309	390538	356511	338216	326747	318873	313127	308747	305297	302509
2.00	523961	427813	390538	370497	357934	349308	343013	338216	334437	331382
2.50	565943	462090	421829	400182	386613	377295	370497	365315	361233	357934
3.00	605018	493996	450954	427813	413306	403346	396078	390538	386174	382647
3.50	641719	523961	478309	453764	438378	427813	420104	414228	409599	405859
4.00	676431	552304	504182	478309	462090	450954	442828	436634	431756	427813

Table 3 : Total Cost within the joined setup EOQ model

TC/TC*	S/s									
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50				1.0000	1.0013	1.0042	1.0078	1.0116	1.0155	1.0193
1.00		1.0000	1.0051	1.0146	1.0250	1.0353	1.0450	1.0541	1.0626	1.0705
1.50		1.0051	1.0206	1.0378	1.0542	1.0695	1.0835	1.0964	1.1082	1.1191
2.00	1.0000	1.0146	1.0378	1.0607	1.0818	1.1010	1.1185	1.1344	1.1489	1.1623
2.50	1.0013	1.0250	1.0542	1.0818	1.1068	1.1293	1.1497	1.1681	1.1850	1.2005
3.00	1.0042	1.0353	1.0695	1.1010	1.1293	1.1547	1.1776	1.1983	1.2172	1.2346
3.50	1.0078	1.0450	1.0835	1.1185	1.1497	1.1776	1.2027	1.2254	1.2462	1.2652
4.00	1.0116	1.0541	1.0964	1.1344	1.1681	1.1983	1.2254	1.2500	1.2724	1.2930

Table 4 : Cost comparison between 'Joined Setup' and 'Optimal' EOQ model

3.3 Two-stage individual EOQ model

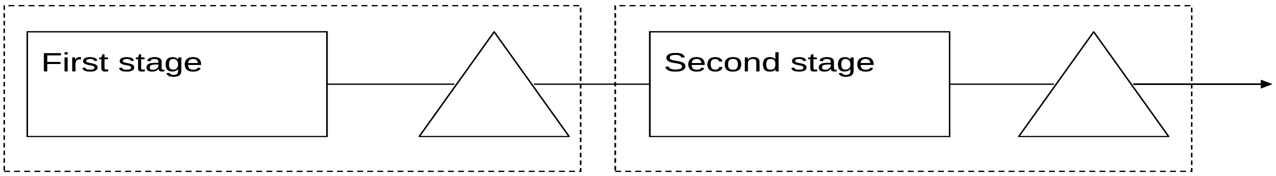

Fig 3 : Two-stage individual EOQ model

Within the 'two stage individual EOQ model' lotsizes for both stages are determined independently, each optimizing their local inventory and change-over costs. Since lotsizes are not synchronised this results in an intermediate stock, acting as a decoupling point. Based on the traditional EOQ logic the lotsizes and total annual cost formulate as :

$$Q=\sqrt{\frac{2.D.S}{h.P}} \qquad M=\frac{D}{Q}=\sqrt{\frac{D.h.P}{2.S}} \qquad q_a=\sqrt{\frac{2.D.s_a}{h.p_a}} \qquad k_a=\frac{Q}{q_a}=\sqrt{\frac{2.D.S.h.p_a}{h.P.2.D.s_a}}=\sqrt{\frac{S.p_a}{s_a.P}}$$

$$TC=M.S+\frac{D}{2M}(1-\frac{1}{k_a})h.P+s_a.k_a.M+\frac{D}{2.k_a.M}.h.p_a$$

Within the sensitivity analysis, total cost is calculated for various s_a and p_a settings (table 5). The cost function of the individually optimised EOQ model remains close to global optimum, when setup costs in both stages have the same magnitude ($S/s_a \approx 1$) and a high added value in second stage (high value for (p_a-P)/P)). The last condition favours to keep material as long a possible in intermediate stock position which corresponds to a decoupled situation. On the condition of large differences in setup ratios (high value of S/s_a) and small product cost differences (low value for (p_a-P)/P) the deviations towards optimal cost raise from 3 to 5% (table 6).

TC	S/s									
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50	376327	317258	291090	275490	264845	256986	250879	245956	241878	238428
1.00	436634	359902	325908	305644	291815	281607	273673	267278	261981	257499
1.50	487084	395575	355035	330869	314377	302203	292741	285115	278797	273453
2.00	531164	426745	380485	352909	334090	320199	309402	300699	293491	287392
2.50	570731	454723	403329	372693	351785	336352	324357	314688	306680	299905
3.00	606900	480298	424211	390777	367960	351118	338027	327476	318736	311342
3.50	640397	503984	443551	407526	382941	364793	350688	339319	329902	321935
4.00	671727	526138	461639	423190	396952	377583	362530	350396	340345	331842

Table 5 : Total Cost within the 'two stage individual EOQ model'

TC/TC*	S/s									
(p-P)/P	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
0.50				1.0516	1.0478	1.0448	1.0424	1.0404	1.0387	1.0372
1.00		1.0303	1.0273	1.0251	1.0235	1.0222	1.0211	1.0202	1.0194	1.0187
1.50		1.0181	1.0164	1.0152	1.0143	1.0136	1.0130	1.0125	1.0120	1.0116
2.00	1.0137	1.0121	1.0111	1.0103	1.0097	1.0093	1.0089	1.0085	1.0082	1.0080
2.50	1.0098	1.0087	1.0080	1.0075	1.0071	1.0068	1.0065	1.0063	1.0061	1.0059
3.00	1.0074	1.0066	1.0061	1.0057	1.0054	1.0052	1.0050	1.0048	1.0047	1.0045
3.50	1.0057	1.0052	1.0048	1.0045	1.0043	1.0041	1.0040	1.0038	1.0037	1.0036
4.00	1.0046	1.0042	1.0039	1.0036	1.0035	1.0033	1.0032	1.0031	1.0030	1.0029

Table 6 : Cost comparison between ‘Two stage individual’ and ‘Optimal’ EOQ model

3.4 Overview and sensitivity study results

Table 7 summarizes for the X-ray filmtype example, the parameter settings and resulting total costs, of the highlighted examples (see table 1 to 6) within the three studied lotsizing models. The sensitivity study reveals that two stage individually optimized EOQ lotsizing should only be used when the end-product stage has a high added value and small setup costs, compared to the first stage (see table 6). The joined setup model, resulting in physically connected operations, should only be used when the end-product stage has a small added value and low change-over costs, compared to the first stage or in the opposite condition of a high end-product added value and high setup cost compared to the first production stage (see table 4).

Model	M	k_a	TC	Cost difference
Optimal two-stage lotsizing model	15.7972	1.7321	376327	
Joined setup model	21.1942	1	390538	1.0378
Two-stage individual EOQ model	15.7972	2.1213	380485	1.0110

Table 7 : X-ray filmtype, model results (single product)

4. First stage (single-product), second stage (multi-product) model

4.1 Optimal two-stage lotsizing model (first stage= single-product, second stage= multi-product)

When extending the single-product towards the multi-product model, the first stage (single-product) intermediate is exploded to a number of end-products, clustered into three end-product clusters (A,B and C). See table 8 for the detailed parametersettings of the X-ray filmtype. The end-product clustered lotsize q_a stands for the sum of n_a lotsizes within the A end-product cluster. The cost of n_a setups corresponds with s_a. The frequency difference factor k_a determines the second stage production frequency equal to k_a.M , with M equal to the first stage production frequency. The annual demand for the intermediate product is the sum of the three end-product cluster demands with $D = d_a + d_b + d_c$. The intermediate product lotsize Q equals D/M and the end-product clustered lotsizes correspond respectively with $q_a=d_a/(k_a.M)$, $q_b=d_b/(k_b.M)$, $q_c=d_c/(k_c.M)$.

Coating Type	Annual Demand (D)	Set-up Cost (S)	Product Cost (P)	Clus Ter	Nr of Products (n_a)	Annual Demand (d_a)	Set-up Cost	Cluster Set-up Cost (s_a)	Product Cost (p_a)
LT2B	17244000	5528	1	A	8	8829000	92.13	737.04	3
				B	14	5518000	92.13	1289.82	3
				C	18	2897000	92.13	1658.35	3
Sum					40	17244000		3685.3	

Table 8 : X-ray filmtype, multi-product parametersettings

Within this logic the total cost function describes as :

$$TC = M.S + M.(s_a.k_a + s_b.k_b + s_c.k_c) + \frac{h.P}{2.M}(D - \frac{d_a}{k_a} - \frac{d_b}{k_b} - \frac{d_c}{k_c}) + \frac{h}{2.M}(\frac{d_a.p_a}{k_a} + \frac{d_b.p_b}{k_b} + \frac{d_c.p_c}{k_c})$$

Using the same logic as within section 3.1 and putting the derivatives to respectively M, k_a, k_b and k_c equal to zero, leads to 4 equations resulting into following optimal values. The provided example of the X-ray filmtype calculates into M = 15.7972, k_a = 2.7713, k_b = 1.6562 and k_c = 1.0583. The total cost corresponds with 362129 euro.

$$M = \sqrt{\frac{D.h.P}{2.S}} \qquad k_a = \sqrt{\frac{d_a.(p_a - P).S}{D.P.s_a}} \qquad k_b = \sqrt{\frac{d_b.(p_b - P).S}{D.P.s_b}} \qquad k_c = \sqrt{\frac{d_c.(p_c - P).S}{D.P.s_c}}$$

4.2 Joined setup model (first stage = single-product, second stage = multi-product)

Within the 'Joined setup model', both stages produce at the same rate (implies k_a = k_b = kc = 1). Since both stages are physically connected there is no intermediate stock. (cfr D-d_a/k_a- d_b/k_b-d_c/k_c = D-d_a -d_b- d_c = 0) The economical production lotsize is controlled by the end-product holding cost and the sum of the first and second stage change-over costs. The provided example of the X-ray coating type calculates into M = 21.1943, k_a = k_b = k_c = 1. The total cost corresponds with 390535 euro. The economical lotsizes and frequency factors formulate as :

$$M = \sqrt{\frac{h.(p_a.d_a + p_b.d_b + p_c.d_c)}{2.(S + s_a + s_b + s_c)}} \qquad k_a = 1 \qquad k_b = 1 \qquad k_c = 1$$

4.3 Two-stage individual EOQ model (first stage = single-product, second stage = multi-product)

Within the 'Two stage individual EOQ model' lotsizes for both stages are determined independently, each optimizing their local inventory and change-over costs. At the first production stage this corresponds with one EOQ lotsize and production rate. At the second stage, lotsizes and production rates are determined for each end-product cluster. Since production rates are not synchronised this results in an intermediate stock, acting as a decoupling point. Based on the traditional EOQ logic the lotsizes and total annual cost formulate as described in table 9. For the X-ray filmtype example total cost corresponds with 365995 euro.

$$TC = M.S + \frac{1}{2M}(D - \frac{d_a}{k_a} - \frac{d_b}{k_b} - \frac{d_c}{k_c})h.P + (s_a.k_a + s_b.k_b + s_c.k_c).M + \frac{h}{2.M}.(\frac{d_a.p_a}{k_a} + \frac{d_b.p_b}{k_b} + \frac{d_c.p_c}{k_c})$$

Coating Type	Annual Demand sqm (D)	EOQ lotsize (Q)	EOQ Fre-quency (M)	Clus Ter	Nr of Products (n_a)	Annual Demand (d_a)	EOQ Lotsize q_i	EOQ Fre-quency m_i	EOQ Freq Factor k_i
LT2B	17244000	1091586	15.7972	A	8	8829000	164663	53.6186	3.3942
				B	14	5518000	172207	32.0428	2.0284
				C	18	2897000	141484	20.4758	1.2962

Table 9 : X-ray filmtype, two-stage individual EOQ model (first phase = single product, second phase = multi-product)

4.4 Comparison of results (first stage = single-product, second stage = multi-product)

Table 10 summarizes for the X-ray filmtype example, the resulting production frequencies and total costs, of the three compared models for a first stage (single-product) and second stage (multi-product) setting.

Model	M	k_a	k_b	k_c	TC	Cost Difference
Optimal two-stage lotsizing model	15.7972	2.7713	1.6562	1.0583	362129	
Joined setup model	21.1942	1	1	1	390535	1.0784
Two stage individual EOQ model	15.7972	3.3942	2.0284	1.2962	365995	1.0107

Table 10 : X-ray filmtype example, model results (first phase = single product, second phase = multi-product)

5. Two-stage multi-product model

When extending the model to a two-stage multi-product setting, the first stage and second stage have respectively M_i and $M_i.k_{ai}$, $M_i.k_{bi}$, $M_i.k_{ci}$ production frequencies. The intermediate products are referenced with the index i, and a,b,c represent their end-product clusters at the second production stage. The numerical example consists of 5 medical film types. The first intermediate product LT2B corresponds with the X-ray example used within the previous sections. Table 11 lists the parameter settings of the studied two-stage multi-product model.

	Coating Type	Annual Demand 1000 sqm (D_i)	Setup Cost € (S_i)	Pro duct Cost €/sqm (P_i)	Clus Ter	Nr of Pro-ducts (n_{ai}),(n_{bi}) (n_{ci})	Annual Demand 1000 sqm (d_{ai}),(d_{bi}), (d_{ci})	Cluster Setup Cost € (s_{ai}),(s_{bi}),(s_{ci})	Product Cost €/sqm (p_{ai}),(p_{bi}),(p_{ci})
1	LT2B	17244	5528	1.00	A	8	8829	737	3.00
					B	14	5518	1290	3.00
					C	18	2897	1658	3.00
2	CX1X5	11500	4240	1.24	A	9	5520	868	3.24
					B	14	4025	1364	3.24
					C	11	1955	1045	3.24
3	CXDGU	10871	8366	2.05	A	10	6088	992	4.05
					B	12	3044	1364	4.05
					C	16	1739	1268	4.05
4	CXDGG	9008	5909	1.86	A	12	4538	744	3.86
					B	24	2737	1488	3.86
					C	18	1733	1180	3.86
5	CXDGL	6348	8738	1.41	A	10	3302	620	3.41
					B	18	1904	1116	3.41
					C	30	1142	1860	3.41

Table 11 : filmtypes example, model parameters (two-stage, multi-product)

5.1 Optimal two-stage lotsizing model

Based on the results on the single-product model (section 3), the optimal first stage production frequency corresponds with the single-stage economical solution. For multiple products lotsize decisions within a production stage, the Doll & Whybark heuristic delivers a still unbeaten near-optimal solution for the ELSP (Economic Lot Scheduling) problem and brings the advantage of a cyclical repetitive production schedule. For the testcase this results in a common cycle repeated 6.5902 (M_{base}) times a year. Four coating types are produced twice ($K_1 = K_2 = K_3 = K_4 = 2$) , The fifth filmtype is coated only once ($K_5 = 1$) within the common cycle. The frequency factors for the second stage (k_{ai}, k_{bi}, k_{ci}) are determined based on the formula presented in section 4.1. Table 12 lists the results.

Index	Intermed. product	M_{base}	K	M	k_a	k_b	k_c	TC
1	LT2B	6.5902	2	13.1804	2.7713	1.6562	1.0583	368084
2	CX1X5	6.5902	2	13.1804	1.9447	1.3247	1.0548	296871
3	CXDGU	6.5902	2	13.1804	2.1466	1.2944	1.0147	397608
4	CXDGG	6.5902	2	13.1804	2.0742	1.1390	1.0178	314485
5	CXDGL	6.5902	1	6.5902	2.1273	1.8251	1.0949	290112
								1667160

Table 12 : Optimal two-stage lotsizing model, production frequencies

Within a operational planning system the proposed cycle length and production frequencies should become rounded to more practical integer values. (M = 6.59 corresponds with a common cycle length of 1 / 6.59 year = 7.79 weeks ≈ 8 weeks).

5.2 Joined setup model

Corresponding with the logic from previous sections 'joined setup' corresponds with an equal production frequency for all products at all stages. When defining this common cycle frequency, the model considers the end-product holding cost and sum of all setup costs. See table 13.

$$M = \sqrt{\frac{\sum_i h.(d_{ai}.p_{ai} + d_{bi}.p_{bi} + d_{ci}.p_{ci})}{2\sum_i (S_i + s_{ai} + s_{bi} + s_{ci})}}$$

Index	Intermediate product	M	k_a	k_b	k_c	TC
1	LT2B	17.3448	1	1	1	398406
2	CX1X5	17.3448	1	1	1	302236
3	CXDGU	17.3448	1	1	1	411034
4	CXDGG	17.3448	1	1	1	322046
5	CXDGL	17.3448	1	1	1	313772
						1747494

Table 13 : Joined setup EOQ model

5.3 Two-stage individual EOQ model

At every stage and for every product(cluster) the optimal production frequency is determined by the standard EQO logic. Table 14 lists the results and the corresponding total cost.

Index	Intermediate product	M	k_a	k_b	k_c	TC
1	LT2B	15.7972	3.3942	2.0284	1.2962	365995
2	CX1X5	16.4029	2.4752	1.6861	1.3425	294317
3	CXDGU	14.5981	3.0546	1.8420	1.4440	405054
4	CXDGG	15.0612	2.8815	1.5824	1.4140	318999
5	CXDGL	9.0525	4.2106	2.3832	1.4297	393616
						1777981

Table 14 : Two-stage individual EOQ model, production frequencies.

5.4 Comparison of results

Model		Total Cost	Difference
Optimal two-stage lotsizing model	First stage (Doll & Whybark procedure) ○ Determine base frequency M_{base} ○ Determine $M_i = K_i.M_{base}$ for each intermediate product Second stage ○ Determine k_{ai} , k_{bi} , k_{ci} for each end-product cluster	1667160	
Joined setup model	First stage & second stage ○ Determine common M for all intermediate products ○ For all end-product clusters $k_{ai} = k_{bi} = k_{ci} = 1$	1747494	1.0482
Two-stage individual EOQ Model	First stage ○ Determine optimal Mi for each intermediate product based on EOQ logic Second stage ○ Determine k_{ai} , k_{bi} , k_{ci} for each end-product cluster based on standard EOQ logic	1777981	1.0665

Table 15 : Comparison of model results

6. Conclusions

This paper proposes for a two-stage multi-product setting, the 'synchronised planning model' realising a global optimum instead of the combination of two locally optimised settings. Within the simplified single-product case, the mathematical model proves (for a two-stage single-product setting) that the optimal two-stage production frequency corresponds with the single EOQ solution for the first stage. A sensitivity study reveals, within these two-stage lotsizing models, the economical cost dependency on product and change-over cost ratio's.

The research reveals that two stage individually optimized EOQ lotsizing should only be used when the end-product stage has a high added value and small change-over costs, compared to the first stage. Physically connected operations (joined setup model) should be used when the end-product stage has a small added value and low change-over costs, or high added value and large change-over costs compared to the first production stage.

Numerical examples prove that for a practical case, the conclusions about the optimal settings remain valid when extending the model to a two-stage multi-product setting. The suggested common cycle approach , based on the economic lot scheduling problem (ELSP) is a practical approach to tackle a two-stage multi-product production and inventory management problem. Within a operational planning system the proposed cycle length and production frequencies should be rounded to more practical integer values.

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