Road Traffic as a Large Network

- Stochastic processes describing internal and external flows of vehicles
- Large network with huge traffic volume has many events that make it difficult to simulate and to control.
- We need an alternative modeling paradigm Stochastic Fluid Model (SFM) that aggregates multiple events
- The objective is to explore the use of SFM for the purpose of control and optimization rather than only for performance analysis.

Flow Models: An Example (Lefeber)

<table>
<thead>
<tr>
<th>Fixed Rate</th>
<th>Variable Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1</td>
<td>A = 5</td>
</tr>
<tr>
<td>B = 2</td>
<td>B = 6</td>
</tr>
</tbody>
</table>

\[
J = \int_0^T [c_1 x_1(t) + c_2 x_2(t)] dt
\]

The Basic SFM: Single class of SFM

Buffer Control and SFM Counterpart

When a vehicle arrives and the queue length goes beyond a given level, it is accepted; otherwise it is rejected.

In Discrete Event System (DES), both \( x(t) \) and \( b \) are integers but in SFM both are treated as real numbers.

Optimization Problems in SFM

\[
J(\theta; x(0), T) = E[f(\theta; x(0), T)]
\]

Where \( f(\theta; x(0), T) \) is a sample function evaluated in the interval \([0, T]\) with initial condition \( x(0) \).

It is difficult to obtain closed-form expression for \( J(\theta, x(0), T) \).

Therefore, we will resort to iterative methods such as stochastic approximation algorithms which are driven by estimates of the cost function gradient w.r.t. the parameter vector of interest.

We seek to obtain \( \theta^{\ast} \) minimizing through an iterative scheme of the form:

\[
\theta_{n+1} = \theta_n - \eta_n H_n(\theta_n; x(0), T, \omega_n), \quad n = 0, 1, \ldots
\]

Where \( H_n(\theta_n; x(0), T, \omega_n) \) is an estimate of \( \frac{dJ}{d\theta} \), evaluated at \( \theta = \theta_n \) and based on information obtained from a sample path denoted by \( \omega_n \).

It is assumed that stationary condition apply to this system. However, we shall consider \( T \) as a fixed time horizon and evaluate performance over \([0, T]\).

We need to estimate \( H_n(\theta_n) \) or \( \frac{dJ}{d\theta} \) and the IPA (Infinite-Semigroup Perturbation Analysis) approach is based on using the sample derivative \( \frac{dJ}{d\theta} \) as an estimate of \( \frac{dJ}{d\theta} \).

The strength of this approach is that \( \frac{dJ}{d\theta} \) can be obtained from observable sample path data alone and can be implemented on line.

Sample Path of SFM

Since solving problem relies on gradient information of given cost function w.r.t. threshold parameters become an essential task, PA methods are therefore suitable if appropriately adapted SFM viewed as a discrete event system.

An event in a sample-path of the SFM can be either exogenous or endogenous and induced event.

An exogenous event is jumping in either \( \{\alpha(t)\} \) or \( \{\beta(t)\} \).

An endogenous event is defined when the buffer becomes full or empty.

An induced event is indicated by \( u_0, u_1, \ldots, u_n \).